IKM - cvičení 1 Skalární částice

$\hat{x}$ poloha - max. sada poz. $\rightarrow$ $|pol:x\rangle$

$\hat{p}$ hybnost - max. sada poz. $\rightarrow$ $|hyb:p\rangle$

$[\hat{x},\hat{p}] = i\hbar\hat{\mathbb{1}}$ $\Leftarrow$

$\hat{x}|pol:x\rangle = x|pol:x\rangle$ $\qquad |pol:\dots\rangle : \mathbb{R} \rightarrow \mathcal{H}$

① $[\hat{x},f(\hat{p})] = i\hbar f'(\hat{p})$ $\Leftarrow$

$$f(p) = \sum_m f_m \hat{p}^m$$

$$[\hat{x},\hat{p}^m] = \hat{x}\hat{p}\dots\hat{p} - \hat{p}\dots\hat{p}\hat{x}$$

$$= \hat{p}\hat{x}\underbrace{\dots\hat{p}}_{m-1} + i\hbar\hat{\mathbb{1}}\underbrace{\hat{p}\dots\hat{p}}_{m-1} - \underbrace{\hat{p}\dots\hat{p}}_{m}\hat{x}$$

$$= \hat{p}\hat{p}\hat{x}\dots\hat{p} + i\hbar\hat{\mathbb{1}}\hat{p}\dots\hat{p} + p\,i\hbar\hat{\mathbb{1}}\dots\hat{p} - \hat{p}\dots\hat{p}\hat{x}$$

$$\vdots$$

$$= \underbrace{\hat{p}\dots\hat{p}}_{m}\hat{x} + i\hbar m\underbrace{\hat{p}\dots\hat{p}}_{m-1} - \underbrace{\hat{p}\dots\hat{p}}_{m}\hat{x}$$

$$= i\hbar m\hat{p}^{m-1}$$ $\leftarrow$ derivace mocniny

linearita derivace $\Rightarrow$ $[\hat{x},f(\hat{p})] = i\hbar f'(\hat{p})$

② $\hat{U}_a = \exp(-\frac{i}{\hbar}a\hat{p})$ $\qquad \hat{U}_a^+ = \hat{U}_{-a}$ $\quad \hat{U}_a\hat{U}_a^+ = \hat{\mathbb{1}}$ $\quad \hat{U}_a\hat{U}_b = \hat{U}_{a+b}$

$$\hat{U}_a\hat{x}\hat{U}_a^+ = ?$$

$$\hat{U}_a\hat{x}\hat{U}_a^+ = \hat{U}_a\hat{U}_a^+\hat{x} + \hat{U}_a[\hat{x},\hat{U}_a^+] =$$

$$= \hat{x} + \hat{U}_a[\hat{x},\exp(\tfrac{i}{\hbar}a\hat{p})]$$

$$= \hat{x} + i\hbar\hat{U}_a\tfrac{i}{\hbar}a\exp(\tfrac{i}{\hbar}a\hat{p}) =$$

$$= \hat{x} - a\hat{\mathbb{1}}$$

③ $\hat{U}_a\hat{x}\hat{U}_a^+ = \hat{x} - a\hat{\mathbb{1}}$ $\Rightarrow$ $\hat{x}\hat{U}_a = \hat{U}_a(\hat{x}+a\hat{\mathbb{1}})$

$$\hat{x}\underbrace{\hat{U}_a|pol:x\rangle} = \hat{U}_a(\hat{x}+a\hat{\mathbb{1}})|pol:x\rangle = (x+a)\underbrace{\hat{U}_a|pol:x\rangle}$$

$\hat{U}_a|pol:x\rangle = |pol:x+a\rangle$ $\leftarrow$ volba fáze báze $|pol:x\rangle$

④ $U_a|pol:x\rangle = |pol:x+a\rangle$ $\quad / \frac{d}{da}$

$-\frac{i}{\hbar}\hat{p}\hat{U}_a|pol:x\rangle = \frac{d}{dx}|pol:x+a\rangle$ $\qquad a=0$

$\hat{p}|pol:x\rangle = i\hbar\frac{d}{dx}|pol:x\rangle$ $\qquad / +\ |stav\rangle$

$\langle pol:x|\hat{p}|stav\rangle = -i\hbar\frac{d}{dx}\langle pol:x|stav\rangle$

$\check{p}\langle pol:x|stav\rangle = -i\hbar\frac{d}{dx}\langle pol:x|stav\rangle$

$\check{p}\,\psi(x) = -i\hbar\frac{d}{dx}\psi(x)$ $\qquad \psi(x) = \langle pol:x|stav\rangle$

x-reprezentace operátoru p

⑤ $\hat{p} \rightarrow \hat{p}|hyb:p\rangle = p|hyb:p\rangle$

$p\langle pol:x|hyb:p\rangle = \langle pol:x|\hat{p}|hyb:p\rangle = -i\hbar\frac{d}{dx}\langle pol:x|hyb:p\rangle$

$\Downarrow$ $\langle pol:x|hyb:p\rangle = c\exp(\frac{i}{\hbar}px)$

normalizace $\langle pol:x|pol:y\rangle = \delta(x|y)$ $\quad \langle hyb\,p|hyb\,q\rangle = \delta(p|q)$

$\Rightarrow c = \frac{1}{\sqrt{2\pi\hbar}}$

$$|hyb:p\rangle = \int_x |pol:x\rangle\langle pol:x|hyb:p\rangle dx = \frac{1}{\sqrt{2\pi\hbar}}\int_x \exp(\tfrac{i}{\hbar}px)|pol:x\rangle$$